FLOW ANALYSIS IN A CHANNEL WITH A MOVING WALL

I. A. Antyshev, S. V. Zhubrin, and V. P. Motulevich UDC 532.542.2

Numerical results obtained by solving the complete system of Navier–Stokes equations for a channel with one movable surface are evaluated.

The problem of thermal and dynamic interaction of a liquid or gas flow with a movable surface is of interest to researchers, especially in recent times when a number of practical applications for this problem have appeared in contemporary technology.

The physical situation characterizing slip of a surface relative to a flow is realized in thermal equipment for continuous processing of sheet material in metallurgy, chemical technology, and the finishing industry, in thermal protection by an indirect draft, in film or suspension cooling, and under flow rarefaction conditions [1-8].

Such flows are among the little-studied class of problems in the theory of motion of a viscous liquid and have unique features [9, 10].

Until the present practically all studies of transport phenomena at movable surfaces have been performed in the boundary-layer approximation. A number of problems have been solved with this approach and certain transport principles and mechanisms in laminar and turbulent boundary layers have been studied [4, 9, 10]. The absence of experimental data for even the simplest cases of laminar flow does not permit conclusions as to the reliability of the theoretical predictions. However, recently performed experiments have essentially confirmed the key features of the theoretical solutions [1, 7, 8].

Symmetrical flows in a planar channel with slippage on the walls produced by flow rarefaction were calculated in [5, 6]. It is obvious that calculation of asymmetric (one wall fixed, the other moving) flows in channels cannot in principle be carried out with boundary layer equations. In such cases the traditional approach based on parabolic-type transport equations can lead not only to significant quantitative errors, but serious qualitative ones as well. Despite this fact, there have been relatively few studies based on more general approaches [11, 12]. Furthermore, those deal with the simplest case of stabilized asymmetric flow in a channel with a moving wall.

The present study will perform a theoretical analysis of the mechanism and principles of momentum transfer in the hydrodynamic stabilization portion of a flow in a planar channel with a moving wall using the complete system of momentum transport equations of various orders

We will consider a model device quite similar to the actual working chambers of apparatus for continuous thermal processing of plane materials (Fig. 1).

The flow of heating agent enters a planar channel bounded below by a fixed wall, and above by the surface of the moving material being processed. Wall boundary layers are formed by the action of viscous forces at the walls. The thickness of these layers increases with removal from the input section until they contact each other. If the channel length is short, then the transport processes will take place under flow development conditions. After the boundary layers contact each other a constant velocity distribution is established, which is independent of x and in the case of an isothermal flow is completely defined by the channel geometry and dynamic state of the system.

Both the developing and the developed hydrodynamically stabilized flow may belong to one of two classes: 1) Poiseuille-type flow with velocity maximum within the flow and friction stresses on the walls of opposite sign ($\tau_2/\tau_1 < 0$); 2) Couette-type flow with continuous velocity change up to a maximum value on the moving wall, in which case the friction stresses on the walls have the same sign ($\tau_2/\tau_1 > 0$). Figure 1 shows the main kinematic features of these flow types.

Moscow Energy Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 4, pp. 592-597, April, 1985. Original article submitted June 8, 1983.

 0022-0841/85/4804-0436$09.50

The efficiency of operation of such a device depends to a significant degree on the character and features of the flow motion, since hydrodynamics exerts the dominant influence on heat–mass transport.

To concentrate our attention upon those physical effects produced by wall movability, we will limit our analysis to the laminar flow regime. We will note that, in this case, carefully performed integration of the transport equations produces sufficiently accurate quantitative results, as compared to physical experiment.

We will make the following assumptions: the flow is isothermal, laminar, and steady state. A plane homogeneous velocity profile is formed at the channel entrance.

With these assumptions the flow of a viscous liquid in a coordinate system attached to the fixed wall is described by Navier–Stokes equations. For a two-dimensional region under steady-state conditions these may be written using the concepts of flow function ψ and vorticity $\omega = \partial v/\partial x - \partial u/\partial y$ in Helmholtz form:

$$\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y}=\frac{1}{\mathrm{Re}}\left[\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right], \tag{1}$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\omega, \tag{2}$$

where the flow function ψ is defined as $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, and all variables with the dimensions of velocity are normalized to U, while length is normalized to H. To complete the mathematical description of the problem, it is necessary to specify boundary conditions in the integration region. At the input the conditions for ψ and ω are calculated from the input profile of longitudinal velocity and the radial velocity is taken equal to zero. On the walls the values of the flow function are constant ($\psi = 0$ on the fixed and $\psi = 1$ on the movable wall). To calculate the vortex intensity near the walls an explicit method and the Woods relationships [13] are used, with a modification for the moving wall. On the movable and fixed surfaces, respectively, we have for the vorticity

$$\omega_N=-3\,(\psi_{N-1}-\psi_N)/(\Delta y)^2-\frac{\omega_{N-1}}{2}-3\,\frac{u_N}{\Delta y},$$

$$\omega_N=-3\,(\psi_{N-1}-\psi_N)/(\Delta y)^2-\frac{\omega_{N-1}}{2}.$$

In the exit plane we impose the condition of constancy along the flow on ψ and ω.

For a stabilized flow the complete momentum transport equations (1) and (2) simplify significantly. In a Cartesian coordinate system for the physical variables

$$\frac{\partial^2 u_d}{\partial y^2}=\frac{H}{\mu U}\frac{\partial P_d}{\partial x}. \tag{3}$$

Eliminating the pressure gradients, after two integrations with consideration of the boundary conditions $y = 0$, $u_d = u_1 = 0$, $y = 1$, $u_d = u_2$, we obtain the velocity distribution

$$u_d=y\left[u_2-6\left(\frac{u_2}{2}-1\right)(1-y)\right]. \tag{4}$$

For the friction coefficients $c_{f_i} = 2\tau_i/\rho U^2$ we have the following expressions:

$$\frac{c_{f_i}}{2}\,\mathrm{Re}=6\left(\frac{u_2}{2}-1\right)(1-2y)-u_2. \tag{5}$$

The corresponding expressions for the relative resistance laws have the simple form

$$S_{2d}=\frac{\tau_2}{\tau_{20}}=1-\frac{2u_2}{3},\quad S_{1d}=\frac{\tau_1}{\tau_{10}}=1-\frac{u_2}{3}. \tag{6}$$

If the channel length is short, it is necessary to calculate flow development in the initial section.

To obtain a solution Eqs. (1) and (2) were reduced to canonical form [14]. In the difference approximation of the problem over the entire solution region, an asymmetric difference grid of first-order accuracy oriented opposite the flow was used. The system of algebraic equations thus obtained was solved by successive Gauss–Zeidel replacement on a nonuniform 21 × 21 grid.

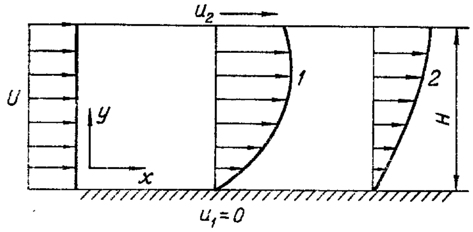

Fig. 1

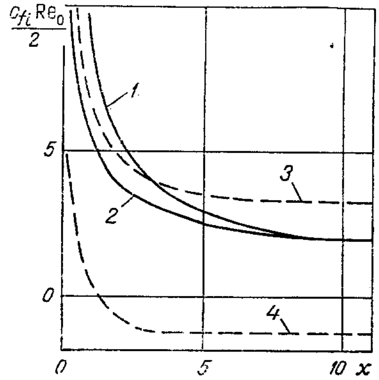

Fig. 2

Fig. 1. Diagram, coordinate system, and main kinematic features of flow: 1) Poiseuille flow; 2) Couette flow.

Fig. 2. Local friction coefficient in initial section of channel with movable wall: 1) u_2, i = 2; 2) 2 and 1; 3) 1.25 and 1; 4) 1.25 and 2.

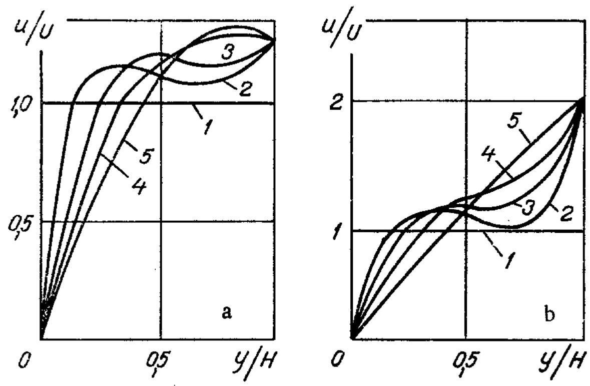

Fig. 3. Distribution of velocity vector components in initial portion of channel at various sections for u_2 = 1.25 (a) and u_2 = 2 (b): 1) x = 0; 2) 0.3; 3) 1; 4) 2.8; 5) 11.7.

Analysis of developing flow calculation in a channel with movable wall (Figs. 2 and 3) reveals two interesting hydrodynamic effects.

For Poiseuille-type flow, in principle, the dependence of the friction coefficient on longitudinal coordinate on the moving wall may change along the flow. The usual monotonic decrease in friction along the surface flowed over is not realized at any velocity. At u_2 = 1, the dependence of friction on longitudinal coordinate is of a nonmonotonic extremal character, and the coordinate of maximum friction ($\tau_2 = 1.05 - 1.2\tau_d$) is practically independent of the velocity of wall motion and Reynolds number Re. This effect can be explained by analysis of the processes affecting the flow near the moving wall. As in the flow in a boundary layer at a moving wall [4], a major role is played by the following factors: first, the difference between velocities of the wall and flow increases friction as it increases; inasmuch as in the given case we are considering development of the flow along the channel, a second factor is the depth to which the perturbing action of the wall penetrates into the flow (the boundary-layer thickness); its value is also affected by the negative pressure gradient which appears due to blockage, a third factor. With increase in boundary-layer thickness friction decreases while the velocity in the flow core increases. We will analyze the physical situation which arises at a wall having a velocity equal to the flow velocity at the channel input with regard to the interaction of these factors.

Directly beyond the input section liquid braking occurs only at the fixed wall, the movable wall offering no resistance to the flow. Gradual blockage of the flow by the boundary layer developing on the fixed surface leads to an increase in flow velocity in the core. A difference develops between the velocities of the flow and movable wall, causing a viscous

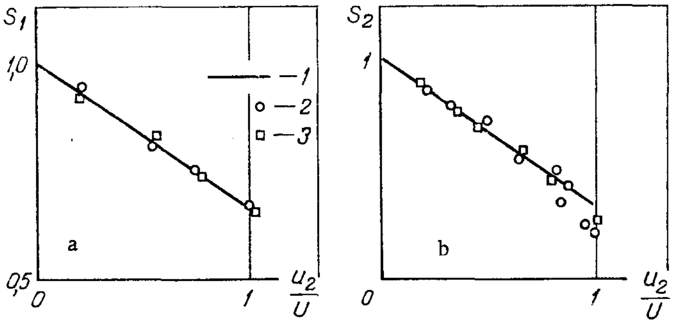

Fig. 4. Relative friction laws in initial channel section on fixed (a) and moving (b) walls: 1) calculation with Eq. (6); 2) $x = 2 \cdot 10^{-3}$; 3) 1. $S_1 = c_{f_1}/c_{f_{10}}$, $S_2 = c_{f_2}/c_{f_{20}}$.

interaction zone is formed. The thickness of this zone increases more rapidly than does the velocity in the flow core. Therefore, having reached some maximum value, the friction then decreases asymptotically, tending to the self-similar value corresponding to a fully developed flow.

If the velocity of the movable wall exceeds the flow velocity in the channel ($u_2 > 1$), then in the range $1 < u_2 < 1.5$ the friction on the wall changes sign as the flow develops (Fig. 2). But this is not caused by breakoff of the flow, and is yet another consequence of the complex nonlinear interaction of the factors enumerated. As is clearly visible from Fig. 3a, as the flow develops the longitudinal velocity profiles transform from characteristic Couette flow bodies (the wall "carries" the flow along) to Poiseuille type with a velocity maximum in the flow. The velocity gradient on the movable wall changes sign while the direction of the flow remains constant.

The development of Couette-type flow ($u_2 > 1.5$) occurs without such singularities in the flow field (Fig. 3b) and on the channel walls. The distribution of the transverse velocity component in the flow also proves to be significantly dependent on the type of flow developing and the mobility of the wall. With increase in velocity of the latter the overall level of motion intensity in the y direction increases significantly.

We will analyze the calculated data on frictional resistance on the channel walls. Figure 4 shows calculation results in relative form. These are compared to relationships for relative friction laws obtained in the developed flow region. Within the limits of the initial section the friction on the walls can be found from Eq. (6). The maximum calculation error over the channel length on the moving wall does not exceed 10% over the range $0 < u_2 \leq 0.6$. For the fixed wall use of the expressions for S_{1d} is valid over the entire range of practical importance $0 < u_2 \leq 1$ with an error of not more than 5%.

Thus, calculation of friction resistance of the developing flow in the channel with a movable wall can be performed by using the known expressions for the initial section of a conventional channel. Correction coefficients can then be calculated from the expressions for relative friction in the developed flow region. We note that although the analysis was performed for a planar two-dimensional flow, special calculations showed that all the major physical features remained unchanged for circular ring channels.

NOTATION

x, y, longitudinal and transverse coordinates; u, v, velocity vector components along x and y axes, respectively; U, mean mass flow velocity; H, channel width; ψ, flow function; ω, vorticity; $Re = UH/\nu$, Reynolds number; $c_f = 2\tau/\rho U^2$, friction coefficient; S, relative resistance coefficient; τ, shear stress. Subscripts: i = 1, 2, fixed and moving channel walls, respectively; 0, flow parameters in channel with fixed wall ($u_2 = 0$); d, fully developed flow region.

LITERATURE CITED

1. V. M. Eroshenko, A. A. Klimov, and L. S. Yanovskii, "Turbulent boundary layer on porous surfaces with drafts at various angles to the wall," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 59-64 (1982).

2. A. R. Wazzan, R. C. Lind, and C. J. Lin, "Laminar boundary layer with mass transfer and slip," Phys. Fluids, 11, No. 6, 1271-1277 (1968).
3. G. R. Inger, "Vectored injection into isobaric laminar boundary layer flows," Warme Stoffubertragung, 5, No. 4, 201-203 (1972).
4. S. V. Zhubrin, V. P. Motulevich, and E. D. Sergievskii, "Gradient flows in a laminar boundary layer at the surface separating immiscible liquids," Tr. Mosk. Energ. Inst., No. 395, 27-34 (1979).
5. E. M. Sparrow, T. S. Lundgreu, and S. H. Lin, "Slip flow in the entrance region of a parallel plate channel," Proc. Heat Transfer and Fluid Mechanics Institute, Stanford (1962), pp. 18-33.
6. B. Gampert, "A Navier—Stokes analysis of developing slip flow," Arch. Mech., 28, 989-996 (1976).
7. E. Bekturganov, K. E. Dzhaugashtin, Z. B. Sakinov, and A. L. Yarin, "Jet flow over a moving surface," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 3, 33-41 (1981).
8. J. S. Tennant and T. Jang, "Turbulent boundary layer flow from stationary to moving surfaces," AIAA J., 11, No. 1, 1156-1160 (1973).
9. G. G. Chernyi, "Boundary layer on a moving surface," in: Selected Problems in Applied Mechanics [in Russian], Nauka, Moscow (1974), pp. 99-104.
10. P. Casal, "Sur l'ensemble des solutions de l'equation de la conche limite," J. Mechanique, 11, No. 3, 459-469 (1972).
11. K. O. Lund and W. Bush, "Asymptotic analysis of plane turbulent Couette—Poiseuille flows," J. Fluid Mech., 96, 81-104 (1980).
12. K. Hanjalic and B. E. Launder, "Fully developed asymmetric flow in a plane channel," J. Fluid Mech., 52, 301-305 (1972).
13. P. J. Roache, Computational Fluid Dynamics, Hermosa (1976).
14. A. Gosmen, V. Pan, A. Ranchel, D. Spolding, and M. Vol'fshtein, Numerical Methods for Solution of Viscous Liquid Dynamics Problems [in Russian], Nauka, Moscow (1972).

HIGH-SPEED FILTRATION REGIMES IN THE MAGNETIC SEPARATION OF PARTICLES FROM LOW-CONCENTRATION MONO- AND POLYDISPERSE SUSPENSIONS OF MAGNETITE

A. V. Sandulyak, O. Yu. Korkhov, V. L. Dakhnenko, and N. N. Kozyar

UDC 532.546.2:66.065.2

A determination is made of high-speed filtration regimes for magnetite suspensions with particles of the same and different coarseness settling in a bed of magnetized balls.

In practice, particle deposition from suspensions takes place in monodisperse and, more frequently, polydisperse suspensions [1], i.e., in suspensions containing particles of roughly the same or different coarseness, respectively. Methods and equipment differing in the nature of the action on the particle — gravitational, centrifugal, electrical, or magnetic — are used, depending on the processing conditions and the properties and coarseness of the disperse-phase particles.

In those cases when the disperse phase of the suspension has ferromagnetic properties, preference is naturally given to magnetic separation. This method is effective, for example, in magnetized granulated media when these suspensions are passed through them. Such media, forming magnetic field-traps characterized by high intensity and a high degree of nonuniformity [2, 3], make it possible to conduct the deposition process at a fairly high rate of suspension filtration: 200-300 m/h [4-8], and even 1000 m/h under favorable conditions [9]. At the same time, it ensures efficient separation of particles of different coarsenesses, in-

Institute of Water Management Engineers, Rovno. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 4, pp. 598-602, April, 1985. Original article submitted February 20, 1984.

 0022-0841/85/4804-0440$09.50